NOAA Earth System
Research Laboratory

THORPEX
A World Weather Research Programme

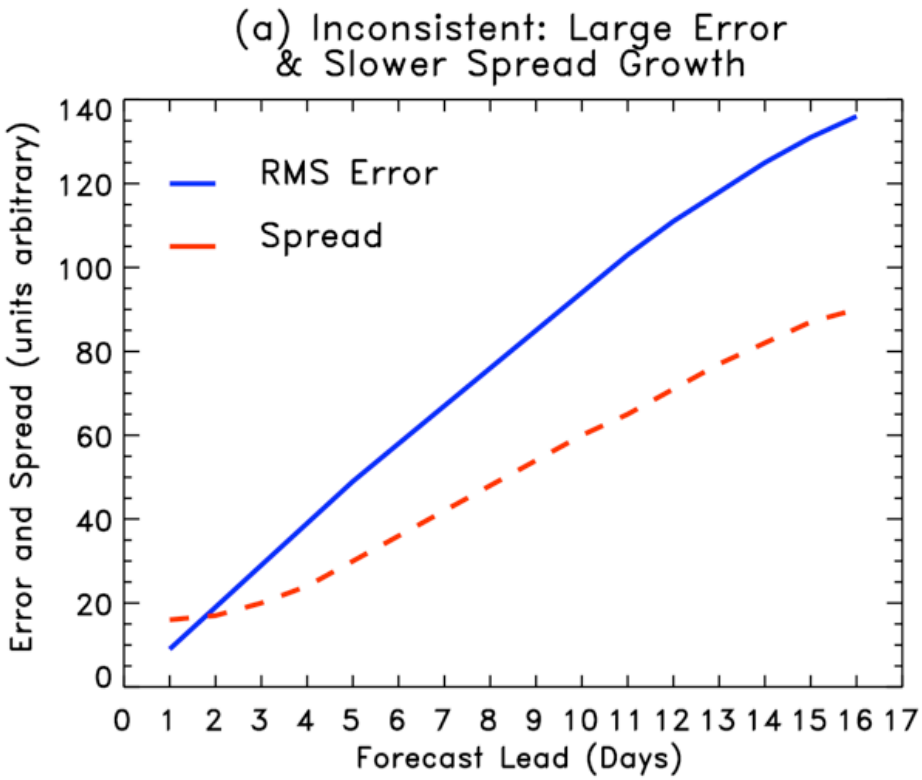
What constrains spread growth in forecasts initialized from ensemble Kalman filters?

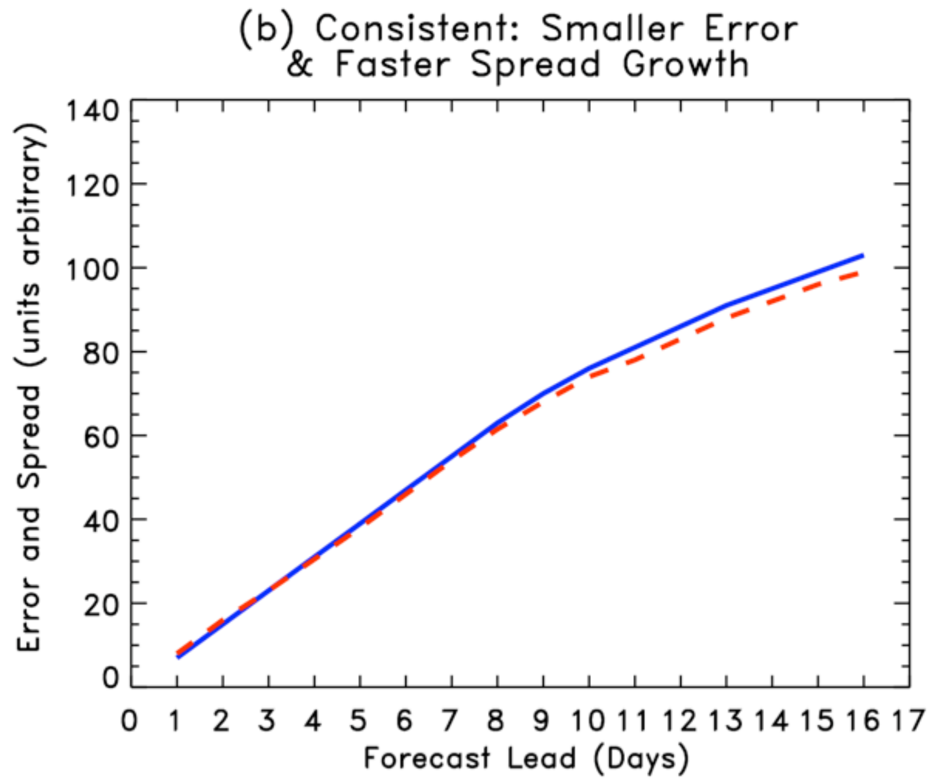
Tom Hamill (& Jeff Whitaker)

NOAA Earth System Research Lab
Boulder, Colorado, USA
tom.hamill@noaa.gov

a presentation to AMS Annual Meeting, December 2010; accepted/major at MWR

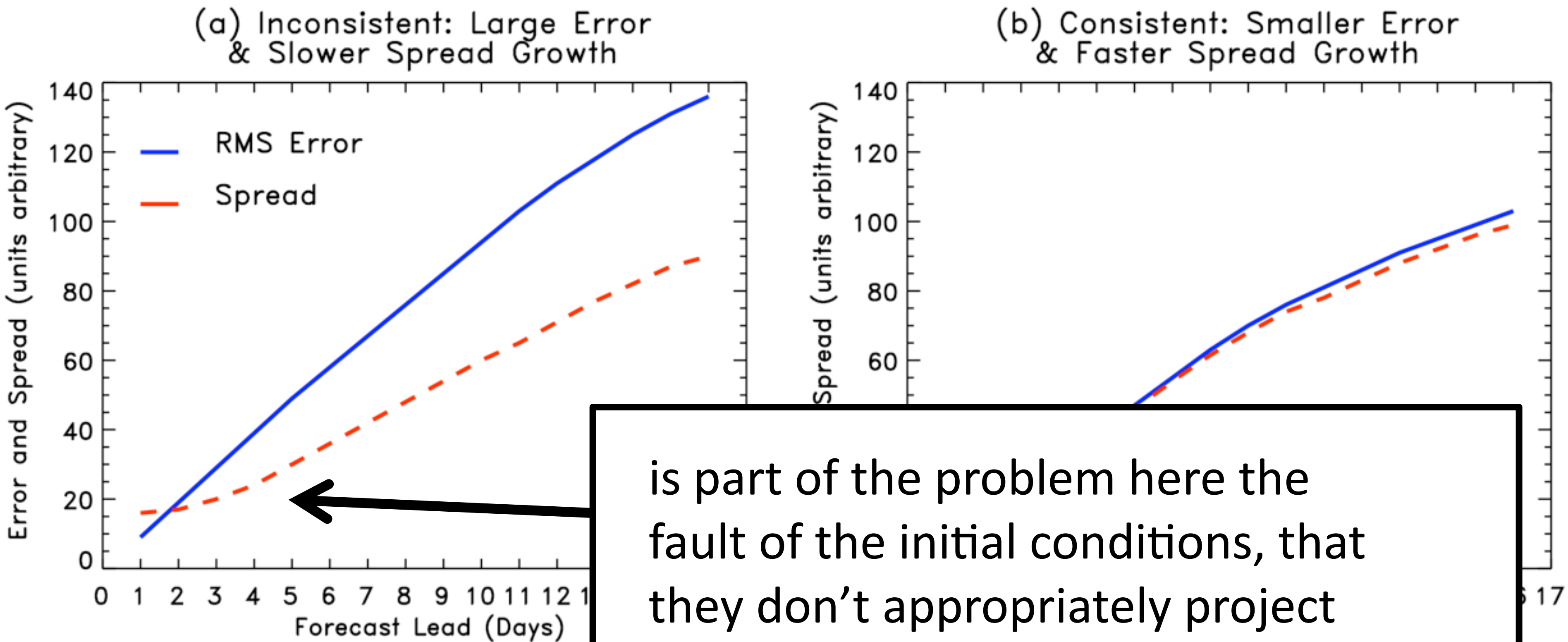
Spread-error consistency





Spread should grow as quickly as error; part of spread growth from manner in which initial conditions are generated, some due to the model (e.g., stochastic physics, higher resolution increases spread growth). **If you don't have this consistency, your ensemble-based probability estimates will be inaccurate**.[2]

Spread-error consistency

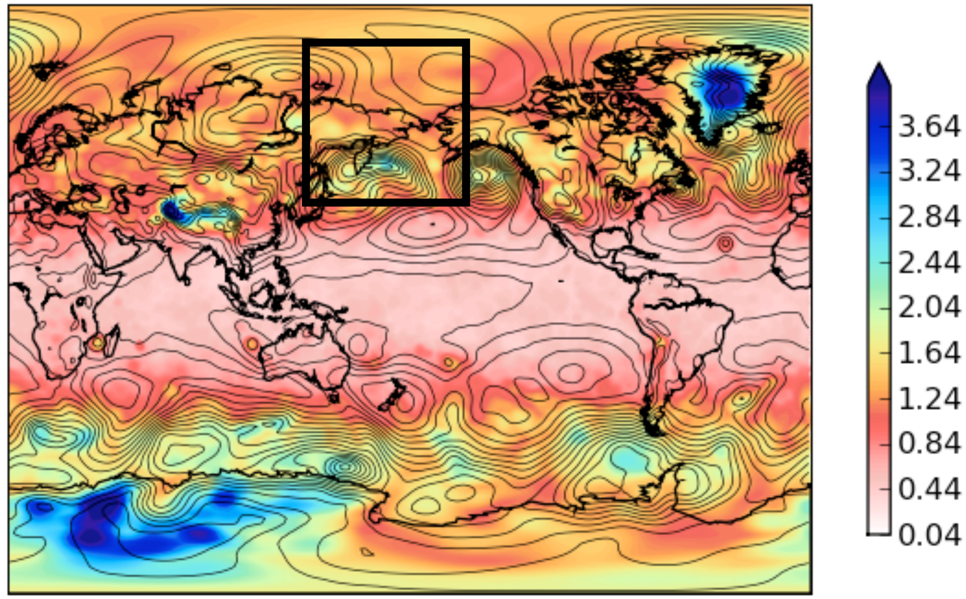


Spread should grow as c
from manner in which initial conditions are generated,
some due to the model (e.g., stochastic physics, higher resolution
increases spread growth). **If you don't have this consistency, your ensemble-based probability estimates will be inaccurate.**

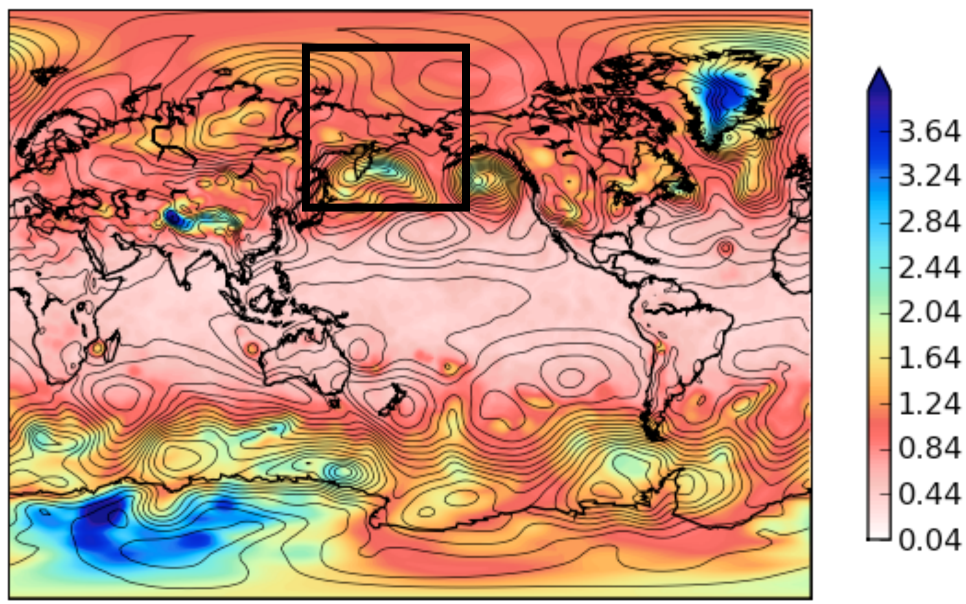
Example: lack of growth of spread in ensemble square-root filter using NCEP GFS

Not much growth of spread in forecast, and decay in many locations. Why?

MSLP analysis spread, 2008-01-01 0600 UTC



First-guess spread 6 h later



Mechanisms that may limit spread growth from ensemble-filter ICs

- Covariance localization used to improve EnKF performance introduces imbalances.
- Method of treating model error (e.g., additive noise) projects onto non-growing structures.
- Model attractor different from nature's attractor; assimilation kicks model from own attractor, transient adjustment process.

Serial EnSRF
("ensemble square-root filter")

$$\overline{\mathbf{x}}^a = \overline{\mathbf{x}}^b + \mathbf{K}\left(\mathbf{y}^o - \mathbf{H}\overline{\mathbf{x}}^b\right)$$

$$\mathbf{K} = \mathbf{P}^b\mathbf{H}^T\left(\mathbf{H}\mathbf{P}^b\mathbf{H}^T + \mathbf{R}\right)^{-1}$$

$$\mathbf{x}_i^{a'} = \mathbf{x}_i^{b'} - \tilde{\mathbf{K}}\mathbf{H}\mathbf{x}_i^{b'}$$

$$\tilde{\mathbf{K}} = \left(1 + \sqrt{\frac{\mathbf{R}}{\mathbf{H}\mathbf{P}^b\mathbf{H}^T + \mathbf{R}}}\right)^{-1}$$

Updates to the mean and perturbations around the mean are handled separately, with "reduced" Kalman gain $\tilde{\mathbf{K}}$ used for perturbations. Rationale in Whitaker and Hamill, 2002 MWR

Methodology

- Apply EnSRF in toy 2-level primitive equation model, examine spread growth (& errors)
  - Perfect-model experiments
  - Imperfect model experiments

- Check a key result in the full NCEP GFS with EnSRF

Toy model, assimilation details

- Assimilation:
  - EnSRF; 50 members.
  - Ensemble forecasts at T31 resolution.
  - Observations: *u*,*v* at 2 levels every 12 h, plus potential temperature at 490 ~ equally spaced locations on geodesic grid. 1.0 m/s and 1.0 K observation errors σ.
- Model: 2-level GCM following Lee and Held (1993) *JAS*
  - T31 resolution for perfect-model experiments; error-doubling time of 2.4 days
  - For imperfect model experiments, T42, with nature run that relaxes to different pole-to-equator temperature difference, different wind damping timescale.

Definitions

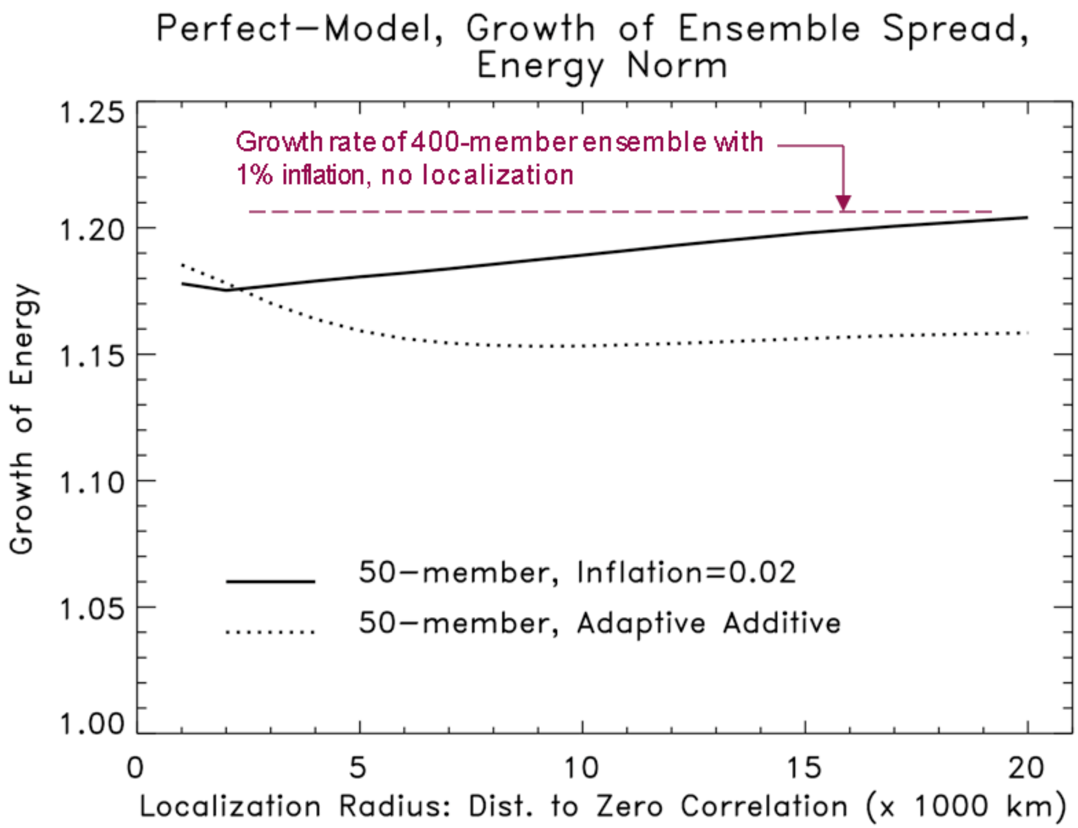
- **Covariance inflation**: $\mathbf{x}_i^b \leftarrow r\left(\mathbf{x}_i^b - \overline{\mathbf{x}}_i^b\right) + \overline{\mathbf{x}}_i^b$

- **Additive noise**: $\mathbf{x}_i^a \leftarrow \mathbf{x}_i^a + \alpha\mathbf{x}_i^n, \quad \alpha\mathbf{x}_i^n \sim N\left(0, \mathbf{Q}\right)$

- **Energy norm**:

$$\| \cdot \| = \frac{\sqrt{\frac{1}{2}\int_A \left[u^2 + v^2 + \frac{c_p}{T_{ref}} T^2 \right] dA}}{\int_A dA}$$

How does spread **growth** change due to localization? (perfect model)



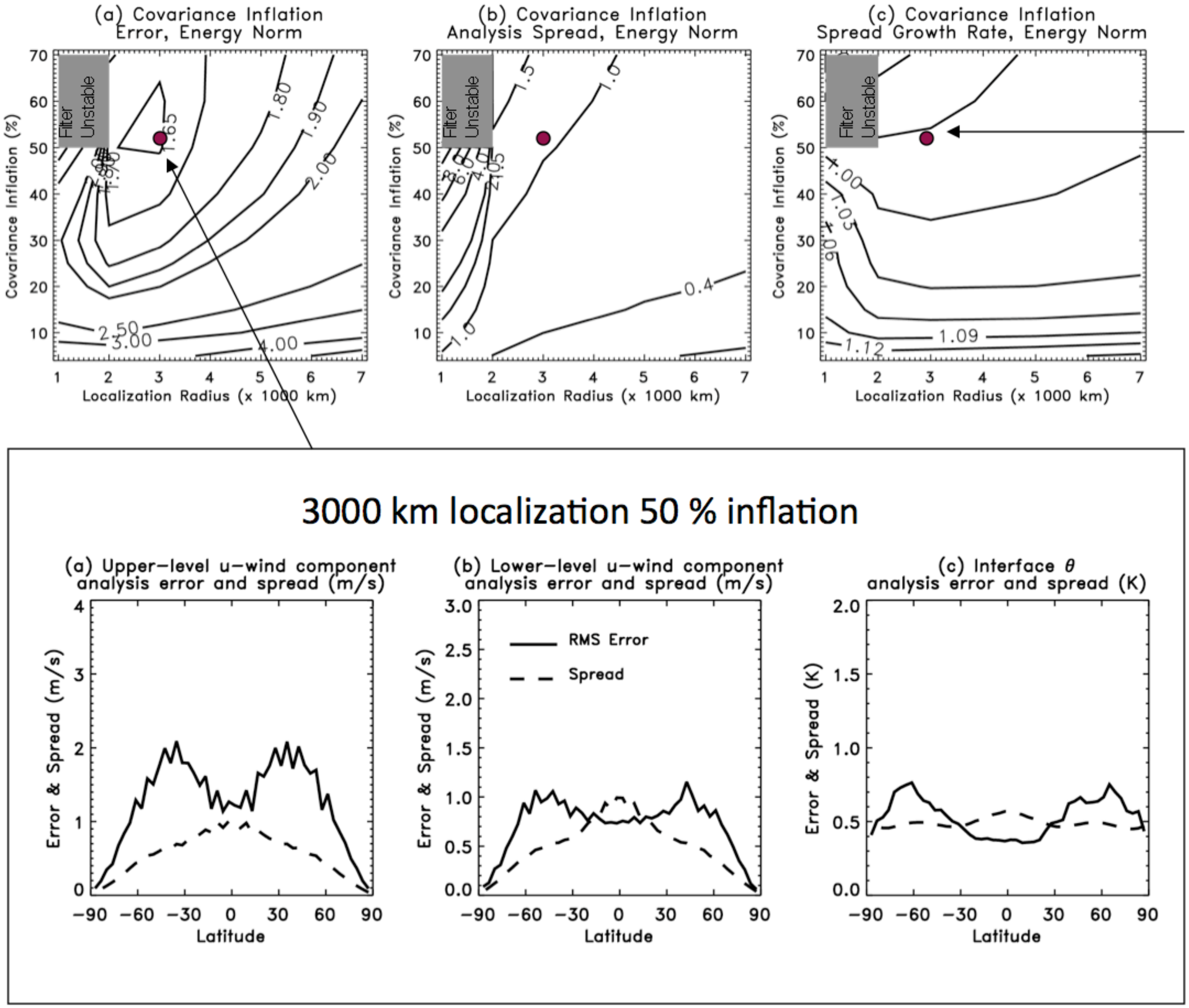
Notes:

(1) Growth rate of 50-member covariance inflation ensemble over 12-h period with large localization radius is close to "optimal"

(2) Increasing the localization radius with constant inflation factor has relatively minor effect on growth of spread. Suggests that in this model, covariance localization is secondary factor in limiting spread growth.

(3) Additive noise reduces spread growth somewhat more than does localization. Adaptive algorithm added virtually no additive noise at small localization radii, then more and more as localization radius increased. Hence, adaptive additive spread doesn't grow as much as localization radius increases because the diminishing imbalances from localization are offset by increasing imbalances from more additive noise.

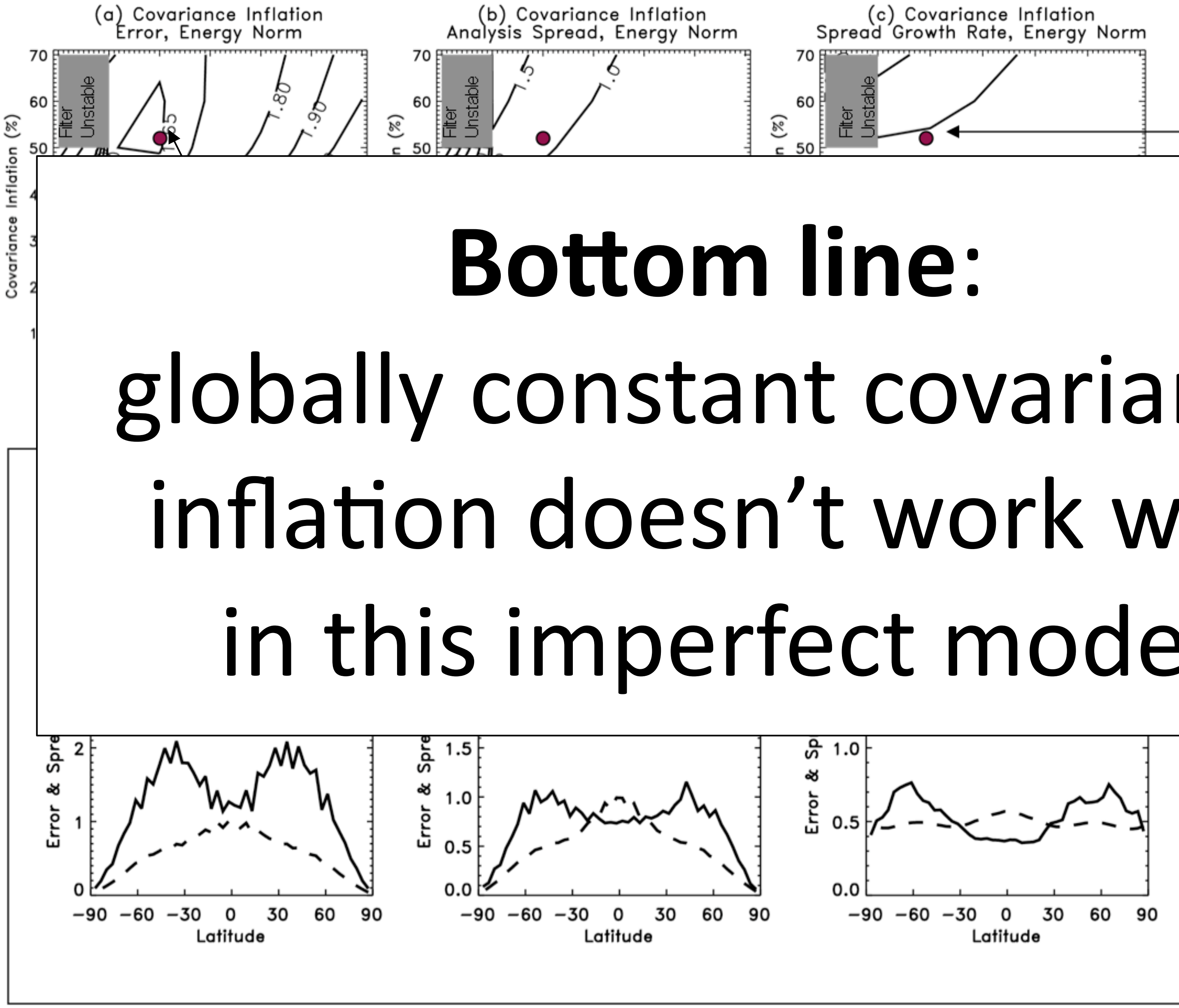
Covariance inflation, **imperfect model**



Spread decays in region of parameter space where analysis error is near its minimum.

Differential growth rates of model error result in difficulties in tuning a globally constant inflation factor (see also Hamill and Whitaker, *MWR*, November 2005)

Covariance inflation, **imperfect model**

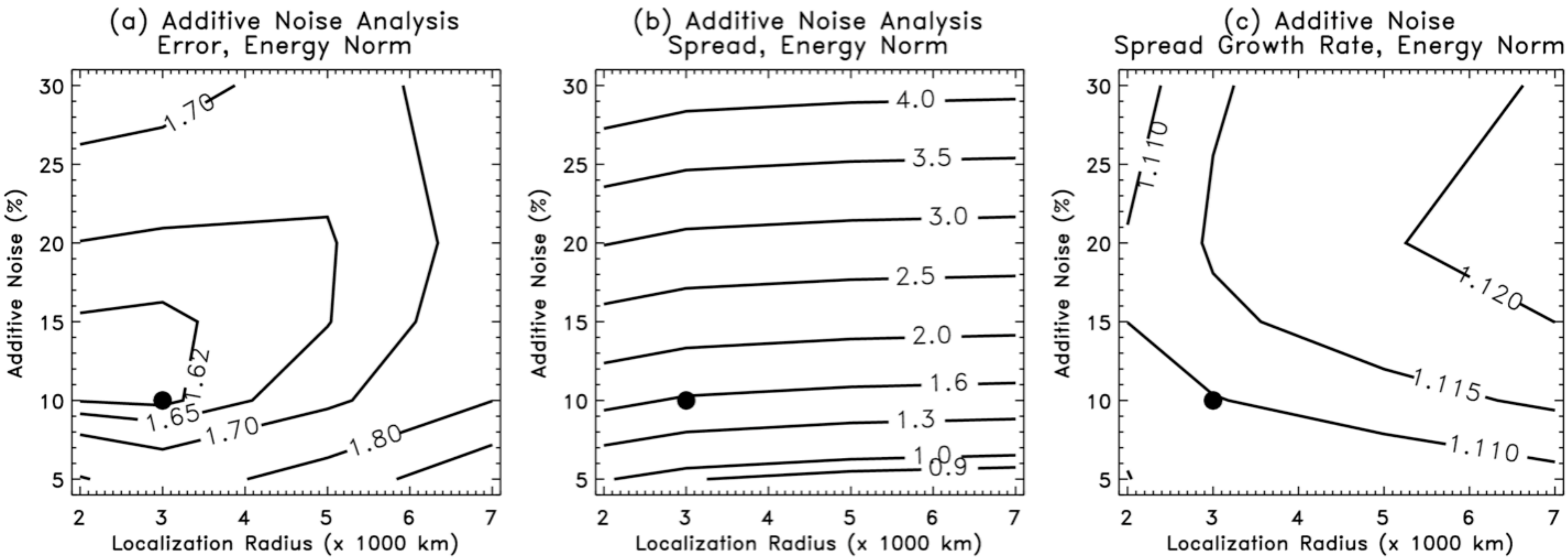


Spread decays in region of parameter
...nalysis
...ts

Bottom line:
globally constant covariance inflation doesn’t work well in this imperfect model

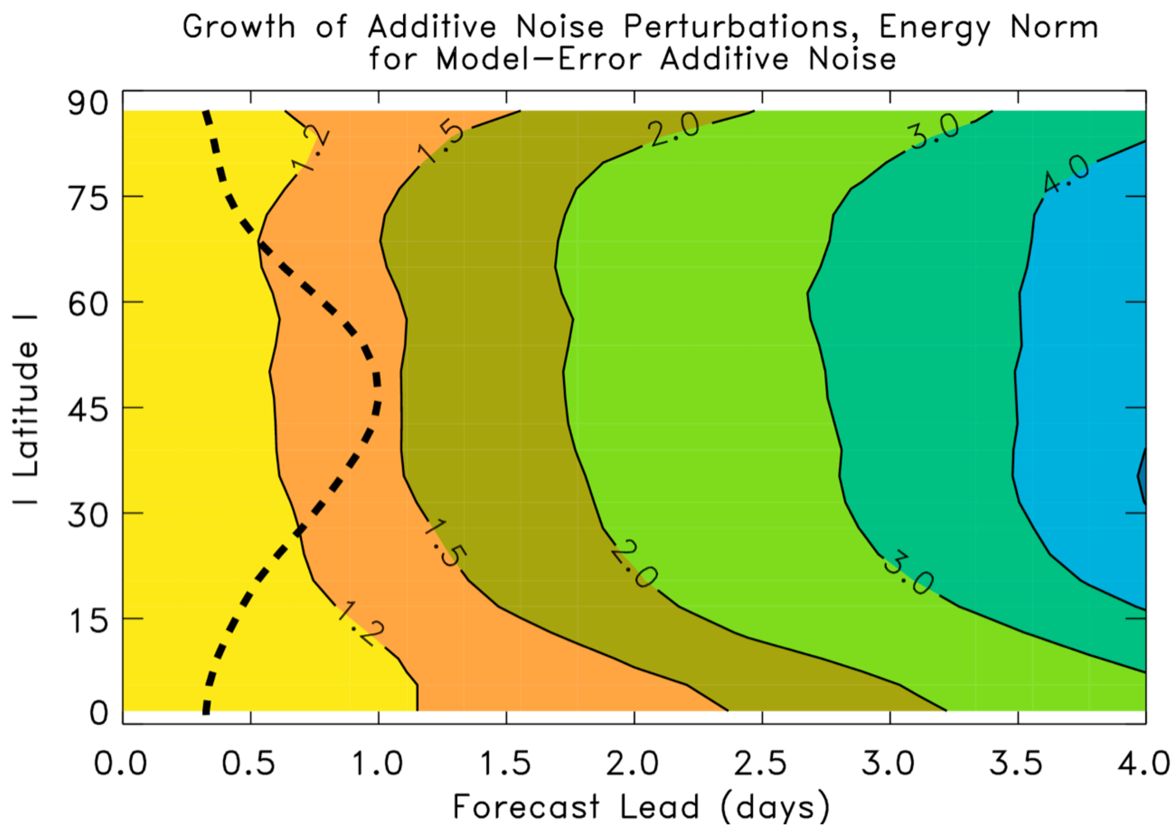
...owth
...l error
...ulties
...bally
...tion
...factor (see also
Hamill and Whitaker, *MWR*, November 2005)

Additive noise, **imperfect model**



Spread growth is smaller than in perfect-model experiments, but is ~ constant over the parameter space.

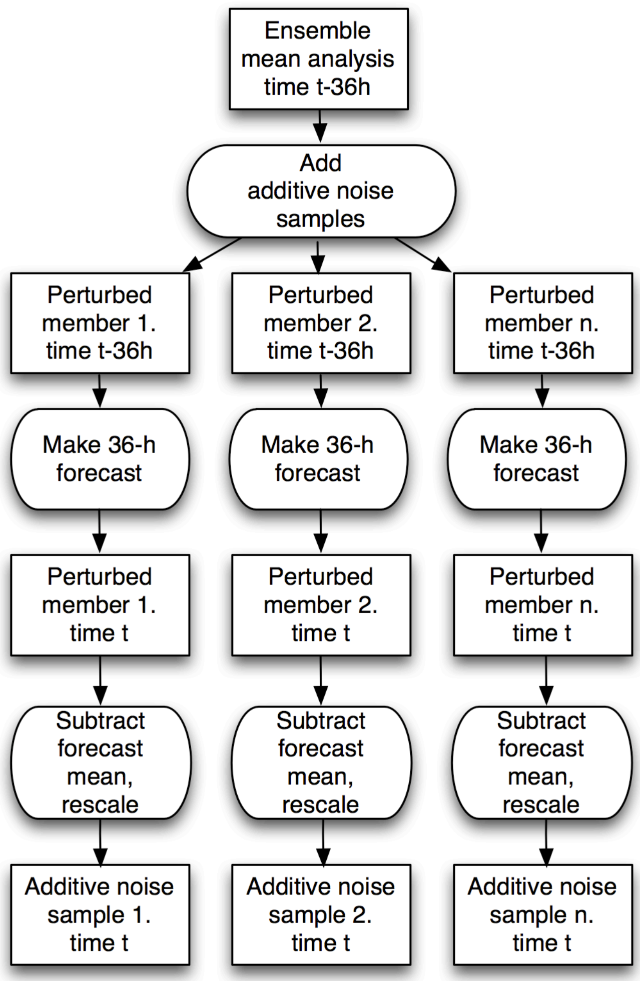
Average growth of additive noise perturbations around nature run



dashed line shows magnitude of initial perturbation

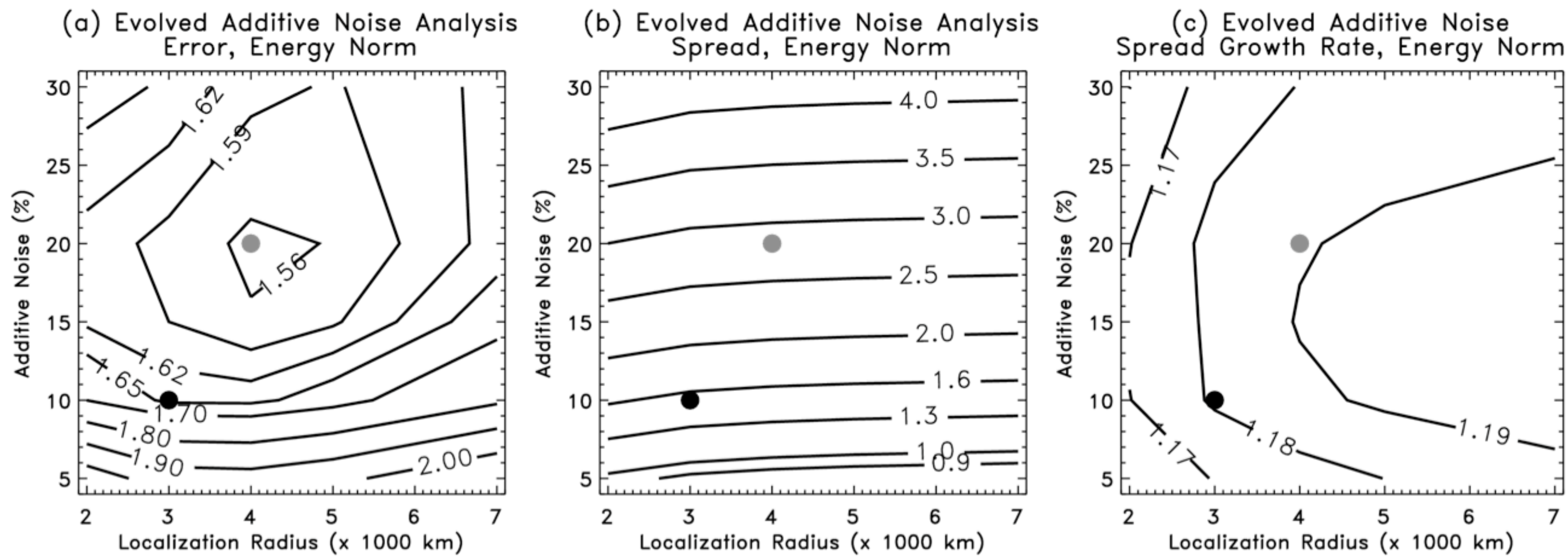
Lesson: it takes a while for the additive noise to begin to project strongly onto system's Lyapunov vectors

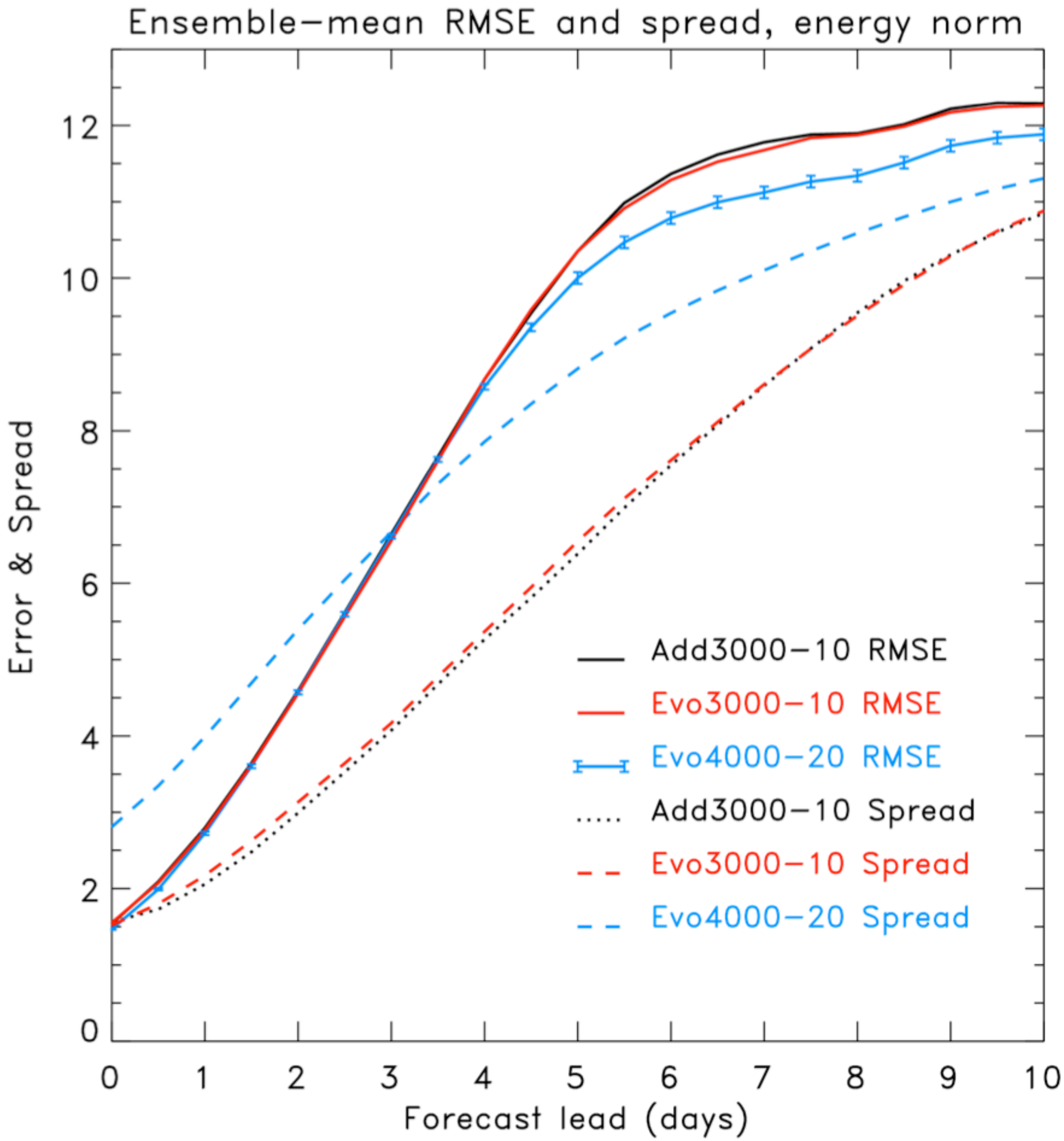
Suppose we evolve the additive noise for 36 h before adding to posterior?



Suppose we evolve the additive noise for 36 h before adding to posterior?

For data assimilation at time *t*, evolved additive error was created by backing up to *t-36* h, generating additive noise, adding this to the ensemble mean analysis at that time, evolving that 36 h forward, rescaling and removing the mean, and adding this to the ensembles of EnKF analyses.



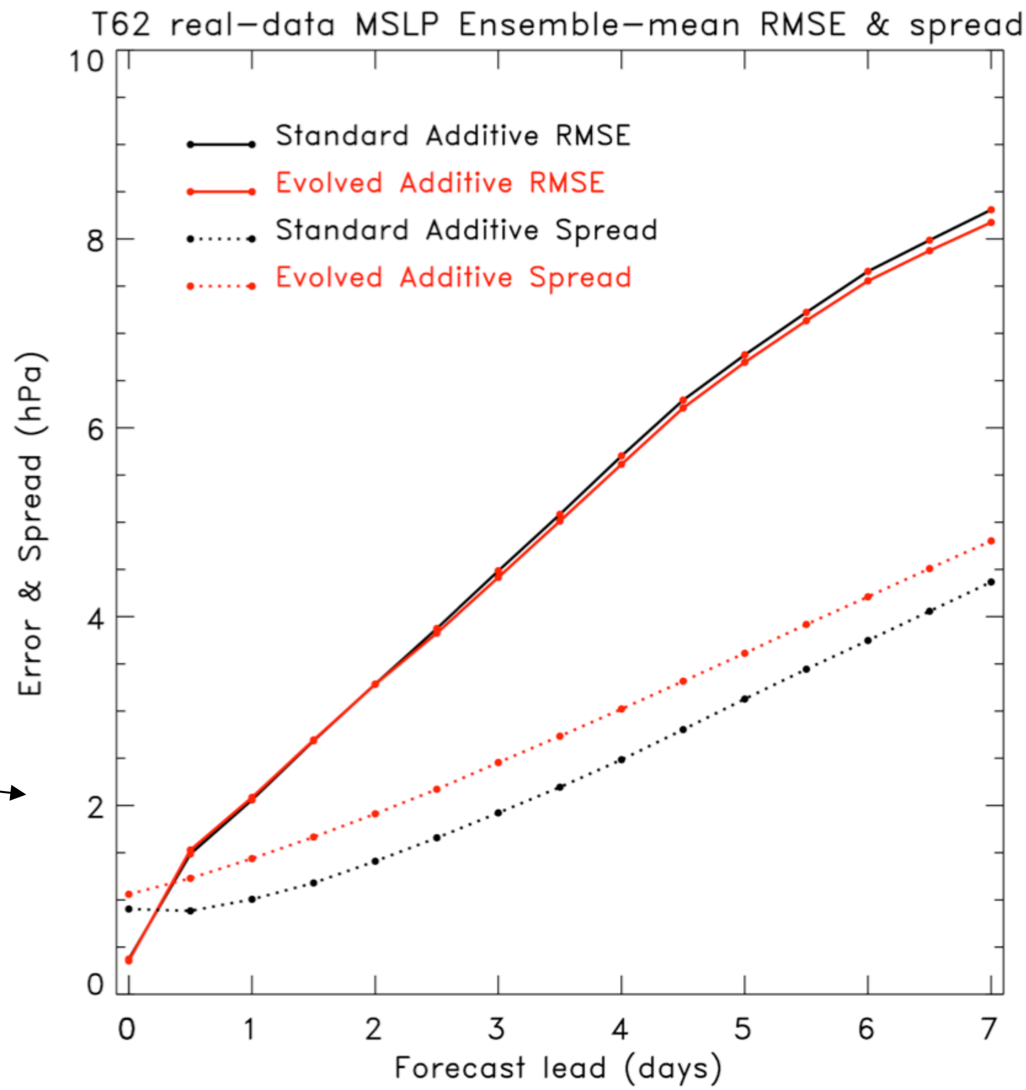


What is the effect on longer-lead ensemble forecasts?

- Not much difference, evolved vs. additive, with same localization / additive noise size.
- An improvement in error, more spread, bigger spread growth with longer localization, more evolved additive noise.

Will results hold with real model, real observations?

- EnKF with T62 NCEP GFS, 10 Dec 2007 to 10 Jan 2008. Nearly full operational data stream.
- 24-h evolved additive error using NMC method (48-24h forecasts) multiplied by 0.5.
- 10-member forecasts 1x daily, from 00Z.
- Main result: slightly higher spread growth at beginning of forecast.
- Other results (T190L64) less encouraging, still being analyzed.



Conclusions

- The non-flow dependent structure of additive noise may be a primary culprit in the lack of spread growth in forecasts from EnKFs.
- Pre-evolving the additive noise used to stabilize the EnKF results in improved spread in the short-term forecasts, and possibly a reduction in ensemble mean error at longer leads.
  - operationally this would increase the cost of the EnKF, but perhaps the evolved additive noise could be done with a lower-resolution model.
- More generally, **the methods to treat system error will affect performance of EnKF for assimilation, ensemble forecasting; require more thought & research**.

Covariance localization & imbalance

$$\mathbf{P}^b = \begin{bmatrix} \sigma^2(u_1) & Cov(u_1,u_n) & Cov(u_1,t_1) & Cov(u_1,t_n) \\ \cdots & & & \\ Cov(u_1,u_n) & \sigma^2(u_n) & Cov(u_n,t_1) & Cov(u_n,t_n) \\ & \cdots & & \\ Cov(u_1,t_1) & Cov(u_n,t_1) & \sigma^2(t_1) & Cov(t_1,t_n) \\ & & \cdots & \\ Cov(u_1,t_n) & Cov(u_n,t_n) & Cov(t_1,t_n) & \sigma^2(t_n) \end{bmatrix}$$

envision a covariance matrix, here with winds and temperatures at *n* grid points

$$\rho = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

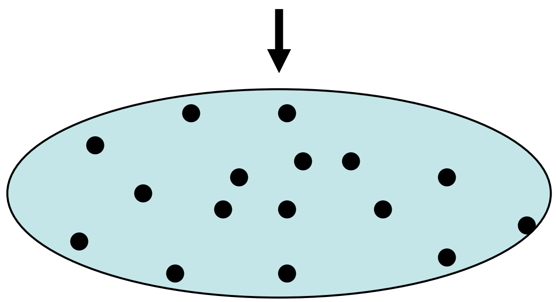
envision a covariance localization at its most extreme, a Dirac delta function, i.e., the identity matrix.

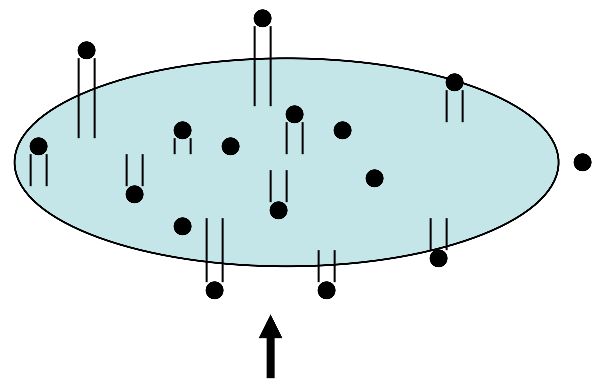
$$\rho \circ \mathbf{P}^b = \begin{bmatrix} \sigma^2(u_1) & 0 & 0 & 0 \\ \cdots & & & \\ 0 & \sigma^2(u_n) & 0 & 0 \\ & \cdots & & \\ 0 & 0 & \sigma^2(t_1) & 0 \\ & & \cdots & \\ 0 & 0 & 0 & \sigma^2(t_n) \end{bmatrix}$$

The localized covariance matrix has totally decoupled any initial balances between winds and temperature

Additive noise

Before additive noise:
ensembles may tend to lie on lower-dimensional attractor

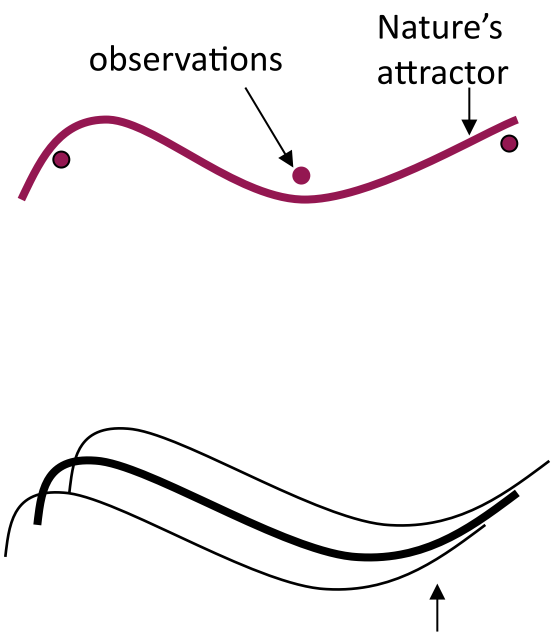




After additive noise:
some of the noise added takes model states off attractor; resulting transient adjustment & spread decay

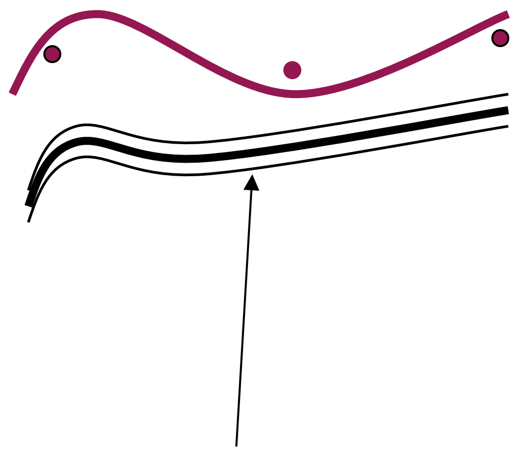
Model error

before data assimilation



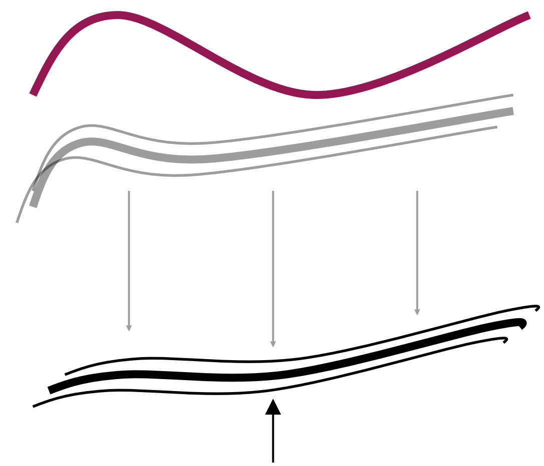
forecast mean
background
and ensemble
members, ~ on
model attractor

after data assimilation



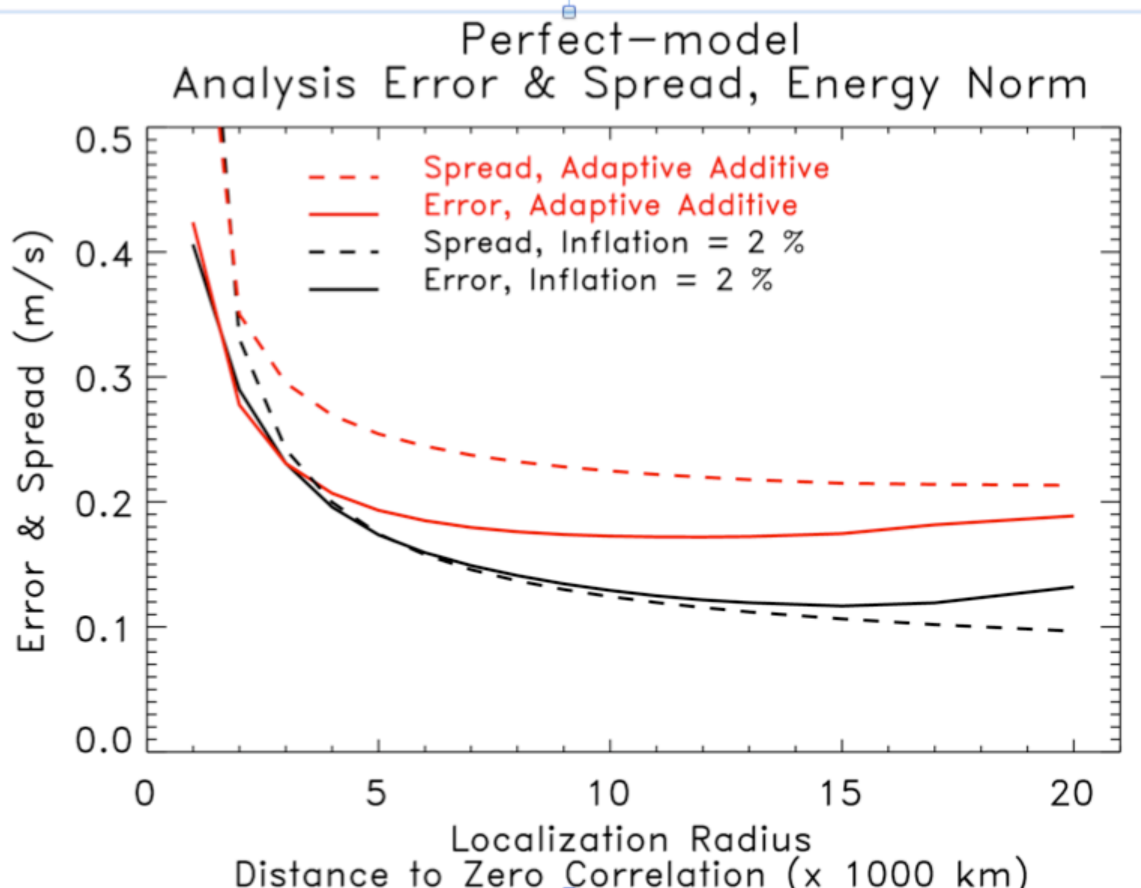
analyzed state,
drawn toward obs;
ensemble (with smaller
spread) off model attractor

after short-range forecasts



forecast states snap
back toward model
attractor; perturbations
between ensemble
members fail to grow.

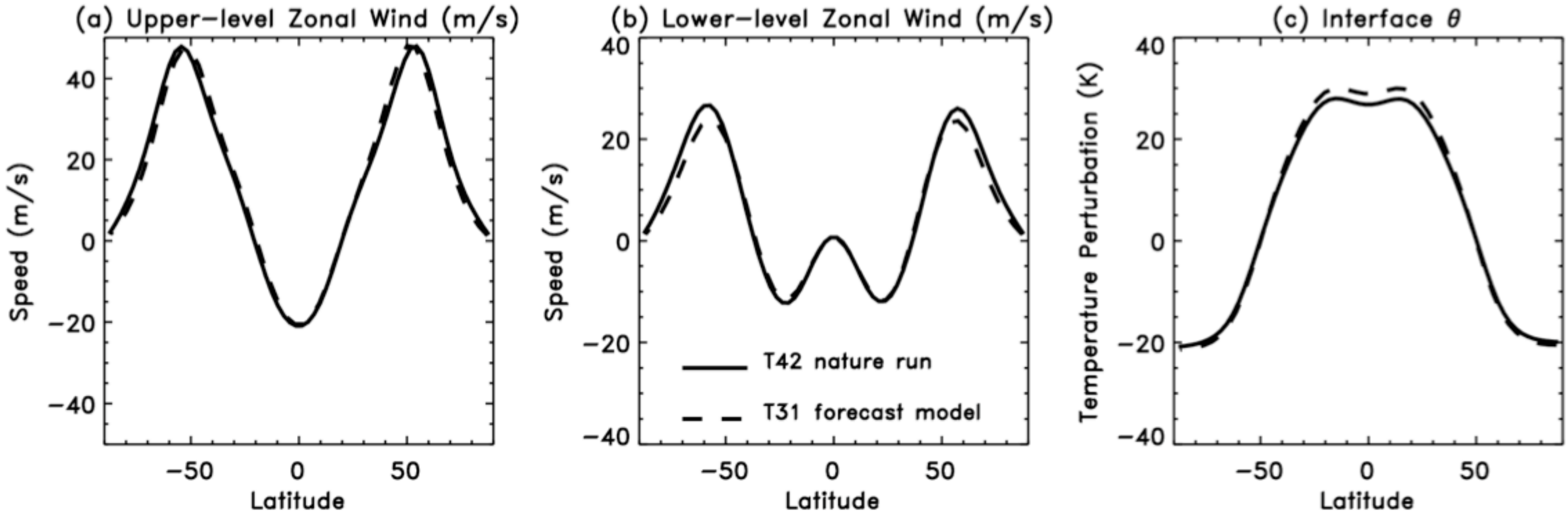
Error/spread as functions of localization length scale, T31 perfect model



Bottom line on errors: for *perfect-model simulation*, covariance inflation is more accurate; deleterious effect of additive random noise.

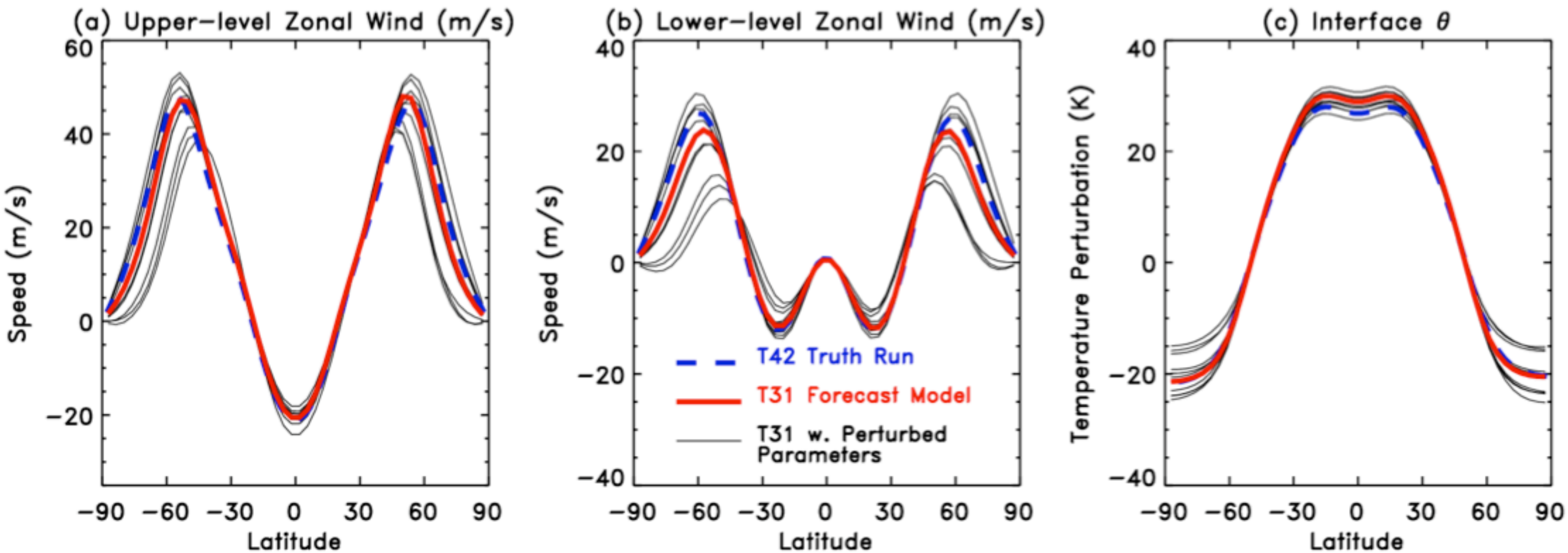
Imperfect-model results:

nature run & imperfect model climatologies



- 6 K less difference in pole-to-equator temperature difference in T42 nature run
- Less surface drag in T42 nature run results in more barotropic jet structure.

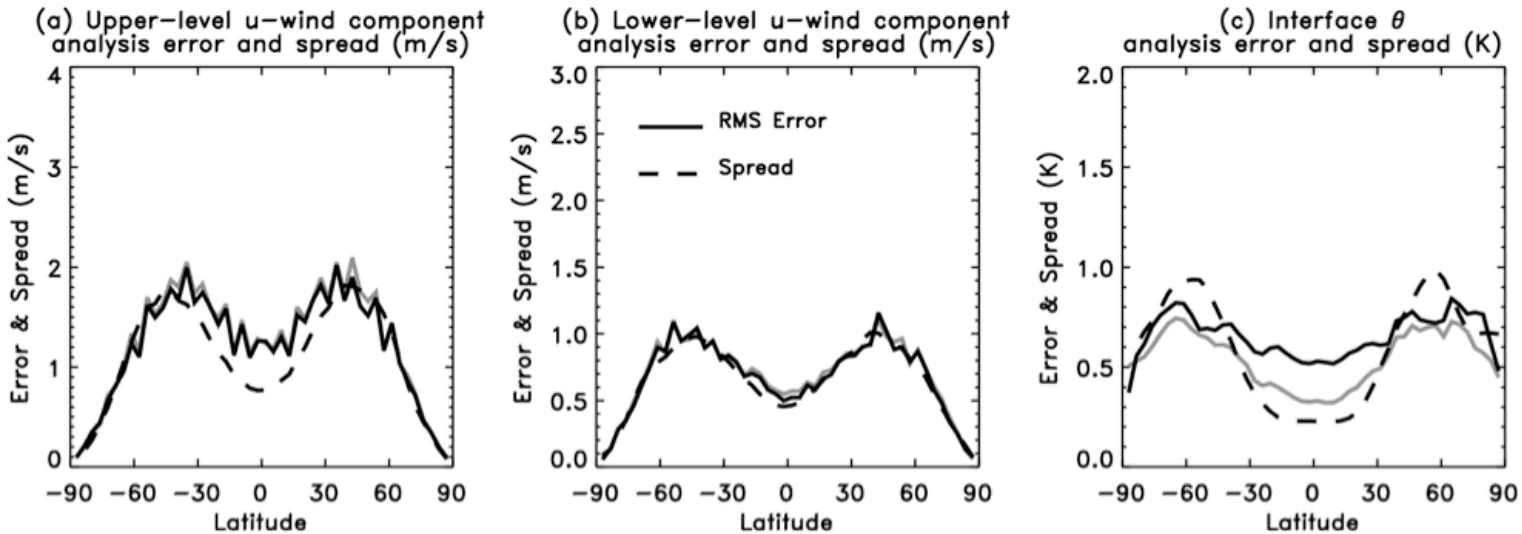
Model error additive noise zonal structure



imperfect-model simulations.

- Additive noise for imperfect model simulations consisted of 50 random samples from nature runs from perturbed models; zero-mean perturbation enforced. 0-24 h tendencies as with perfect model did not work well given substantial model error.

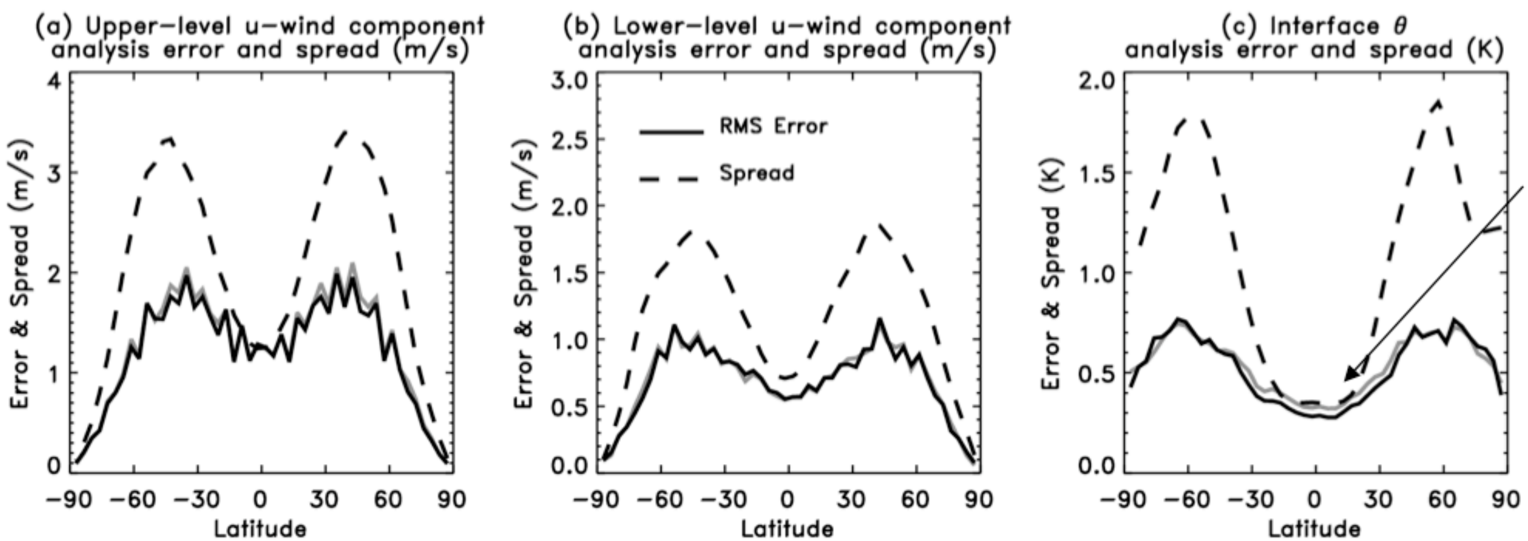
- Evolved, 3000 km localization, 10% inflation



grey line is error result from non-evolved additive noise (replicated from slide 12)

higher error in tropics, less spread than error.

- Evolved, 4000 km localization, 20% inflation



now slightly reduced error in tropics, much greater spread than error.